Describe plant by root, shoot masses m_r, m_s, uptake rate v, tissue nutrient content f_n as fraction of mass

Growth rate limited by nutrient uptake rate, v $\dot{m}^{ul} = m_r v / \tilde{f}_n$

Formulate as relative growth rate $RGR^{ul} = \frac{m_r v}{(m_r + m_s)\tilde{f}_n} = \frac{r v}{(1+r)\tilde{f}_n}$

Carbon gain: PS rate per leaf area is N-use efficiency, p, times N content, f_n; a fraction α_L of shoot mass is leaf mass; efficiency of converting photosynthate to biomass is β*

Growth rate limited by C gain: $\dot{m}^{pl} = \beta p^* f_n \alpha_L m_s$

Formulate as relative growth rate $RGR^{pl} = \beta p^* \alpha_L \frac{m_s}{m_s + m_r} f_n = \beta p^* \alpha_L f_n / (1+r)$

The two relative growth rates must be equal on average → this is a condition on f_n; here is the solution for f_n:

$$r v / f_n = \beta p^* \alpha_L f_n$$

$$\rightarrow f_n = \sqrt{r v / (\beta p^* \alpha_L)}$$

This gives us the relative growth rate, then: $RGR = \frac{\sqrt{r v}}{1+r} \sqrt{\beta p^* \alpha_L}$