

Relatively far from the individual root, the concentration becomes nearly asymptotic to a “bulk-soil” value, c_b :

$$c_b = c_a \left(\frac{r}{b} \right)^{-k} + \frac{j_a a}{kD} \left[1 - \left(\frac{r}{b} \right)^{-k} \right] \quad (6)$$

Combine with equation for uptake rate determined by root physiology:

$$j_a = V_{\max} \frac{c_a}{c_a + K_m} \quad (7)$$

Obtain a quadratic equation for concentration at root surface, c_a :

$$c_a = \frac{-g + \sqrt{g^2 + 4fc_b K_m}}{2f}, \text{ with } f = \left(\frac{b}{a} \right)^{-k},$$
$$g = fK_m + x - c_b, \text{ and } x = \frac{V_{\max}}{v_a} \left[1 - \left(\frac{b}{a} \right)^{-k} \right] \quad (8)$$

Plug the numerical value of c_a into (7) to estimate achieved uptake rate, j_a .

Conclusion from numerical results (even correcting approximations here):
(Mass flow) – (its suppression of diffusion) \rightarrow no significant change in uptake