Diffusion combined with mass flow; local concentration in soil solution is c = c(r) (cylindrical symmetry most places); v = inflow velocity; neglect adsorption (as for nitrate) or fold into D approximately

Uptake rate j as determined by the soil physics is then $$j = vc + D\frac{dc}{dr} \quad (1)$$

In quasi-steady state, flux crossing any radius r is same as at any other r, including the radius at the root surface, a:

$$j(r)r = j(a)a \rightarrow j = \frac{j_a a}{r};\ \ similarly, v = \frac{v_a a}{r} \quad (2)$$

Rewrite the diffusion equation $$D\frac{dc}{dr} = -\frac{v_a a}{r}c + \frac{j_a a}{r} \quad (3)$$

Solve by adjoint method: $$c(r) = c_a\left(\frac{r}{a}\right)^{-k} + \frac{j_a a}{kD}\left[1-\left(\frac{r}{a}\right)^{-k}\right]$$ *with* $$k = \frac{v_a a}{D} \quad (4)$$

Compare with limiting case of v=0 (no mass flow)

$$c(r) = c_a + \frac{j_a a}{D}\ln\left(\frac{r}{a}\right) \quad (5)$$